# THE MONTHLY PREDICTION EXPERIMENTS USING A COUPLED ANALOGY-DYNAMICAL MODEL

Huong Jianping (黄建平) and Wang Shaowu (王绍武)

Department of Geophysics, Peking University, Beijing

Received January 3, 1990

#### ABSTRACT

Considering from point of view of the dynamics, it is convenient to regard the field to be predicted as a small disturbance superposed on the historical analogous field, and thus the statistical technique can be used in combining with the dynamics. Along this line, a coupled atmosphere-earth surface analogy-dynamical model is formulated and applied to making monthly prediction.

This approach facilitated the utility of the useful information contained in both the historical data set and the initial field to improve the dynamic model based solo on the latter and show better skill in prediction.

Key words: long-range prediction, dynamical prediction, historical analogy, analogy-dynamical approach

## I. INTRODUCTION

In recent years, considering extensive and serious social and economic impacts of climatic anomalies, the long-range forecasting and prediction are paid more attention not only by meteorologists, but also by governments. The studies on the prediction technique of climatic anomalies and the physical factor in their formation are of greatly theoretical and practical significance. Unfortunately, the operational techniques of long-range forecast are mainly based on the statistics and experiences. Therefore, a numerical prediction model was designed with considering the dynamics, and sucking the experiences in operational long-range forecast into the model.

The use of analogues has a long history in meteorology. In long-range prediction the technique consists of examining historical data, identifying one or more months or seasons that resemble the month or season just past and predicting that the following month or season just past will resemble that observed in the period following the chosen "analogue" (Nicholls, 1984). This phenomenon is often used in operational forecast (Wang, 1984) and called as historical analogous approach. However, the historical analogous approach imagines the future to be a simple repetition of the history. It limits greatly the forecast skill.

Considering from point of view of the dynamics, it is convenient to regard the field to be predicted as a small disturbance superposed on the historical analogous field, and thus the statistical technique can be used in combining with the dynamics. Along this line, a coupled atmosphere-earth surface analogy-dynamical model is formulated and applied to making monthly prediction,

#### II. FORMULATION OF THE ANALOGY-DYNAMICAL MODEL

# 1. Prediction Equation of Atmosphere

The analogy-dynamical model is a coupled atmosphere-earth surface model which includes two parts, i.e. the atmosphere and the earth surface. The governing equations for atmosphere are vorticity equation and diabatic thermodynamical equation, and the controlling equation for the earth's surface is the heat conduction equation for surface temperature, in which the thermal advection by the current in the oceanic area is taken into account. The interaction between atmosphere and surface is accomplished through such physical processes as turbulence, radiation, condensation, etc. (Chao et al., 1982).

Because the evolution of the meteorological fields starting from similar state (i.e. initial and boundary conditions) always keeps such similarity running in a certain period, the evolution of monthly mean circulation can be regarded as a small disturbance superposed on the historical analogous field, the ocean and atmospheric states can be divided into basic state and disturbance state (Huang, 1988; 1989), i.e.

$$X = oldsymbol{ ilde{X}} + oldsymbol{\hat{X}}$$
 ,

where basic state  $\tilde{X}$  is the monthly mean value of a historical analogous year that is selected from historical data according to similarity criterion and its evolution has already been known.  $\hat{X}$  is the disturbance state, which is the difference of two analogous years' state and called as analogous deviation (Huang, 1988; 1989). It is assumed that the basic state satisfies the basic equation and substituted  $X = \tilde{X} + \hat{X}$  into the basic equation, the analogous deviation equations can be written as

$$\frac{\partial}{\partial t} \nabla^2 \hat{\phi} + \frac{1}{f} J(\hat{\phi}, \nabla^2 \hat{\phi}) + \frac{1}{f} J(\hat{\phi}, \nabla^2 \hat{\phi}) + \frac{1}{f} J(\hat{\phi}, \nabla^2 \hat{\phi}) + \frac{2\Omega}{a^2} \frac{\partial \hat{\phi}}{\partial \lambda}$$
$$= f^2 \frac{\partial \hat{\omega}}{\partial p} - \alpha \nabla^2 \hat{\phi} , \qquad (1)$$

$$\frac{\partial}{\partial t} \left( \frac{\partial \hat{\phi}}{\partial p} \right) + \frac{1}{f} J \left( \tilde{\phi}, \frac{\partial \hat{\phi}}{\partial p} \right) + \frac{1}{f} J \left( \hat{\phi}, \frac{\partial \hat{\phi}}{\partial p} \right) + J \left( \hat{\phi}, \frac{\partial \hat{\phi}}{\partial p} \right) + \sigma_p \hat{\omega} = -\frac{R}{p} \hat{Q}, \qquad (2)$$

where

$$\nabla^{2} = \frac{1}{a^{2}} \left[ \frac{1}{\sin^{2}\theta} \frac{\partial^{2}}{\partial\lambda^{2}} + \frac{1}{\sin\theta} \frac{\partial}{\partial\theta} \sin\theta \frac{\partial}{\partial\theta} \right],$$
$$J(A,B) = \frac{1}{a^{2}\sin\theta} \left[ \frac{\partial A}{\partial\theta} \frac{\partial \nabla^{2} B}{\partial\lambda} - \frac{\partial A}{\partial\lambda} \frac{\partial \nabla^{2} B}{\partial\theta} \right],$$

 $\phi$  the geopotential height, f the Coriolis parameter,  $\sigma_P = (R^2 T/P^2 g) (\gamma_d - \gamma)$  the static stability; the deviation of diabatic heating rate  $\hat{Q} = \hat{\epsilon}/(\rho c_P)$  is as follows:

$$\frac{\hat{\varepsilon}}{\rho c_{P}} = \frac{\hat{\varepsilon}_{s}}{\rho c_{P}} + \frac{\hat{\varepsilon}_{R}}{\rho c_{P}} + \frac{\hat{\varepsilon}_{L}}{\rho c_{P}}, \qquad (3)$$

where  $\hat{\epsilon}_{s}/(\rho c_{p})$  denotes the deviation of the turbulent heat exchange and is taken as

$$\frac{\hat{\boldsymbol{\varepsilon}}_s}{\rho \boldsymbol{\varepsilon}_p} = \frac{\partial}{\partial p} k_p \frac{\partial \hat{\boldsymbol{T}}}{\partial p}, \qquad (4)$$

where  $k_p = \rho^2 g^2 k_T$ ,  $k_T$  is the coefficient of heat conductivity due to turbulence, g is the gravity. The deviation  $\hat{\boldsymbol{\varepsilon}}_R/(\rho \boldsymbol{c}_P)$  of the radiation heat exchange includes solar short wave radiation which is simply expressed by an empirical formula and atmospheric long wave radiation which is given by the scheme of Kuo (1968). It can be written as

$$\frac{\hat{\boldsymbol{\varepsilon}}_{R}}{\rho \boldsymbol{c}_{P}} = \frac{I}{\rho \boldsymbol{c}_{P}} \left(1 - \boldsymbol{c}_{s} \hat{\boldsymbol{n}}\right) \left(1 - \boldsymbol{\alpha}_{s}\right) + \frac{\partial}{\partial p} \boldsymbol{k}_{R} \frac{\partial \hat{\boldsymbol{T}}}{\partial p} - \frac{\hat{\boldsymbol{T}}}{\boldsymbol{\tau}_{R}}, \qquad (5)$$

where  $\bar{I}$  the solar radiation,  $c_s$  the empirical constant,  $\alpha_s$  the earth's albedo, the deviation of cloudiness  $\hat{n}$  is proportional to  $\hat{\omega}_b$  which is the deviation of the vertical velocity on the top of boundary layer, i.e.

$$\hat{n} = \frac{\widehat{\omega}_b}{\widetilde{\omega}_0}, \qquad (6)$$

where  $\tilde{\omega}_0$  is an empirical parameter  $\hat{\omega}_b = l_b \nabla^2 \hat{\phi}_s / f$ ,  $l_b = \sqrt{k_T/2f}$  is the thickness of the boundary layer and  $\nabla^2 \hat{\phi}_s$  the geostropic vorticity on the earth's surface. For the deviation of the heat exchange of condensation  $\hat{e}_L/(\rho c)_p$ , it may be simply parameterized (Chao, 1982) as

$$\frac{\hat{\varepsilon}_L}{\rho c_P} = -\frac{L}{c_P} \frac{d\hat{q}}{dt} \approx \frac{L}{c_P f} \frac{d\ln \tilde{e}_s}{d\tilde{T}} l_b \tilde{q}_s(p) \nabla^2 \hat{\phi}_s, \qquad (7)$$

where  $\tilde{q}_s$  is saturated specific humidity, and L is the latent heat.

The model atmosphere is firstly divided into three vertical layers in P coordinates, i.e. 300, 500 and 700 hPa which are indicated by numerals 1, 2 and 3, respectively. The boundary condition and difference may be written as

$$p = 0, \quad \hat{\omega} = 0, \quad \left(\frac{\partial \phi}{\partial p}\right)_{0} = 0,$$
$$p = P_{s}, \quad \hat{\omega} = 0, \quad \left(\frac{\partial \phi}{\partial p}\right)_{4} = -\frac{R}{P_{s}}\hat{T}_{s},$$

and

$$\begin{pmatrix} \frac{\partial \hat{\phi}}{\partial p} \end{pmatrix}_{1} = \frac{1}{\sqrt{p}} (\hat{\phi}_{2} - \hat{\phi}_{1}) , \left( \frac{\partial \tilde{\phi}}{\partial p} \right)_{1} = -\frac{R}{P_{1}} \tilde{T}_{1},$$

$$\begin{pmatrix} \frac{\partial \hat{\phi}}{\partial p} \end{pmatrix}_{2} = \frac{1}{2\sqrt{p}} (\hat{\phi}_{3} - \hat{\phi}_{1}) , \left( \frac{\partial \tilde{\phi}}{\partial p} \right)_{2} = -\frac{R}{P_{2}} \tilde{T}_{2},$$

$$\begin{pmatrix} \frac{\partial \hat{\phi}}{\partial p} \end{pmatrix}_{3} = \frac{1}{\sqrt{p}} (\hat{\phi}_{3} - \hat{\phi}_{2}) , \left( \frac{\partial \tilde{\phi}}{\partial p} \right)_{3} = -\frac{R}{P_{3}} \tilde{T}_{3}.$$

Because the anomaly field of monthly mean geopotential height has the equivalent barotopic structure (Huang, 1988; 1989), the deviation of geopotential height at 300hPa, 700hPa and surface can be expressed in terms of the deviation of 500 hPa geopotential height, i.e.  $\hat{\phi}_1 = B_1 \hat{\phi}_2$ ,  $\hat{\phi}_3 = B_3 \hat{\phi}_2$ ,  $\hat{\phi}_s = B_4 \hat{\phi}_2$ , where  $B_i$  (*i*=1,3,4) is the regression coefficient. Thus, the vertical difference of deviation term

can be written as

$$\left(\frac{\partial\hat{\phi}}{\partial p}\right)_{1} = E_{1}\hat{\phi}_{2}, \qquad E_{1} = \frac{1}{\varDelta p}\left(1 - B_{1}\right),$$

$$\left(\frac{\partial\hat{\phi}}{\partial p}\right)_{2} = E_{2}\hat{\phi}_{2}, \qquad E_{2} = \frac{1}{2\varDelta p}\left(B_{3} - B_{1}\right),$$

$$\left(\frac{\partial\hat{\phi}}{\partial p}\right)_{2} = E_{3}\hat{\phi}_{2}, \qquad E_{3} = \frac{1}{\varDelta p}\left(B_{3} - 1\right).$$

Finally, by eliminating  $\hat{\omega}$  from Eqs. (1) and (2), the vertical difference of the geopotential deviation tendency equation has the form:

$$\frac{\partial}{\partial t} (\nabla^2 \hat{\phi}_2 - \lambda \hat{\phi}_2) = -\mathscr{L} (\hat{\phi}_2) - \alpha \nabla^2 \hat{\phi}_2 + Q_{41} \hat{\phi}_2 + Q_{42} \nabla^2 \hat{\phi}_2 + Q_{51} \hat{T}_5, \qquad (8)$$

where

$$\begin{aligned} \mathscr{L}(\hat{\phi}_{2}) &= \frac{1}{f} \left[ J(\tilde{\phi}_{2}, \nabla^{2} \hat{\phi}_{2}) + J(\hat{\phi}_{2}, \nabla^{2} \tilde{\phi}_{2}) + J(\hat{\phi}_{2}, \nabla^{2} \tilde{\phi}_{2}) \right] \\ &+ \frac{2\Omega}{a^{2}} \frac{\partial \hat{\phi}}{\partial \lambda} + J(A_{1} \tilde{\phi}_{3} - A_{2} \tilde{\phi}_{1}, \hat{\phi}_{2}) + J(A_{3} \tilde{T}_{3} - A_{4} \tilde{T}_{1}, \hat{\phi}_{2}), \\ \lambda &= \frac{f^{2}}{2\Delta p} \left[ \frac{E_{1}}{\sigma_{p1}} - \frac{E_{3}}{\sigma_{p3}} \right], \\ A_{1} &= \frac{fE_{3}}{2\Delta p \sigma_{p3}}, A_{2} = \frac{fE_{1}}{2\Delta p \sigma_{p1}}, \\ A_{3} &= \frac{fB_{3}R}{2\Delta p \sigma_{p3}P_{3}}, A_{4} = \frac{fB_{1}R}{2\Delta p \sigma_{p1}P_{1}}, \end{aligned}$$

 $\hat{\phi}_2$  is the geopotential height deviation at 500 hPa,  $\tilde{\phi}_1, \tilde{\phi}_2$  and  $\tilde{\phi}_3$  are the monthly mean geopotential height of the historical analogous year at 300, 500 and 700 hPa, respectively.  $\tilde{T}_1$  and  $\tilde{T}_3$  are the monthly mean temperature at 300 and 700 hPa, respectively. The difference approach is used for numerical solution, and time filtering scheme are used for time integration with a 4-hour time increment.

## 2. Prediction Equation of Earth's Surface Temperature

The surface temperature deviation equation and the corresponding vertical boundary conditions are as follows

$$\frac{\partial \hat{T}_{s}}{\partial t} + \delta [J_{s}(\tilde{\psi}_{s}, \hat{T}_{s}) + J_{s}(\hat{\psi}_{s}, \tilde{T}_{s}) + J_{s}(\hat{\psi}_{s}, \hat{T}_{s})] = k_{s} \frac{\partial^{2} \hat{T}_{s}}{\partial z^{2}}, \qquad (9)$$

$$z=0, \rho_{s}c_{Ps}k_{s}\left(\frac{\partial\hat{T}_{s}}{\partial z}\right)-\rho c_{P}k_{T}\left(\frac{\partial\hat{T}}{\partial z}\right)+\delta\left(\rho Lk_{T}\gamma \frac{d\ln\tilde{e}_{s}}{d\tilde{T}}\frac{\partial\tilde{q}_{s}}{\partial\tilde{T}}\right)\hat{T}_{s}=-\frac{S_{0}}{\tilde{W}_{0}f}l_{b}B_{4}V^{2}\hat{\phi}_{2},$$

$$z = -D, \ T_s = 0,$$
$$J_s(A,B) = \frac{\sqrt{2}}{2} \frac{0.0126}{\sqrt{\cos\theta}a^2} \left[ \left( \frac{\partial A}{\partial \theta} + \frac{1}{\sin\theta} \frac{\partial A}{\partial \theta} \right) \frac{1}{\sin\theta} \frac{\partial B}{\partial \lambda} - \left( \frac{\partial A}{\partial \theta} - \frac{1}{\sin\theta} \frac{\partial A}{\partial \theta} \right) \frac{\partial B}{\partial \theta} \right],$$

where  $\delta = 1$  for ocean and  $\delta = 0$  for land,  $\psi_s$  is the stream function of ocean current. The

boundary condition is deduced from the energy balance equation for the surface.

The solution of boundary value problem (Navarra and Miyakoda, 1984) can be found according to the theory of differential equation. The solution of surface temperature deviation for the time step  $t + \Delta t$  has the following analytical form

$$\hat{T}_{s}^{t+\Delta t} = S_{1}\hat{T}_{s}^{t} + S_{2}H_{1}^{t} + S_{3}H_{2}^{t} + S_{4}\nabla^{2}\hat{\phi}_{2}^{t}, \qquad (10)$$

where the time step  $\Delta t$  is taken as 24 hours,  $H_1$  is the thermal advection by occanic current and  $H_2$  is the vertical average of the thermal advection in the atmosphere. The right hand side terms represent the effects of persistence, oceanic thermal advection, atmospheric thermal advection and short wave radiation deviation adjusted by cloudiness deviation, respectively:  $S_1, S_2, S_3$  and  $S_4$  are the corresponding influence coefficients in the following forms

$$\begin{split} H_{1} &= \frac{\partial}{k_{s}} [J(\tilde{\psi}_{s}, \hat{T}_{s}) + J(\hat{\varphi}_{s}, \tilde{T}_{s} + \hat{T}_{s})], \\ H_{2} &= \frac{1}{kf_{0}} \bigg[ J\left(\tilde{\varphi}_{2}, -\frac{E_{2}P_{2}}{R}\hat{\varphi}_{2}\right) + J(\hat{\varphi}_{2}, \tilde{T}_{2}) \bigg], \\ S_{1} &= D \bigg( 1 + \frac{\rho c_{P}}{\rho_{s} c_{Ps}} \sqrt{\frac{k_{T} \tau_{R}}{k_{s} \varDelta t (\varDelta t + \tau_{R})}} \bigg), \\ S_{2} &= -Dk_{s} \varDelta t, \\ S_{3} &= -D \frac{\rho c_{P} k_{T} \varDelta t}{\rho_{s} c_{Ps}} \sqrt{\frac{\tau_{R}}{k_{s} (\varDelta t + \tau_{R})}}, \\ S_{4} &= -\frac{DS_{0} \sqrt{k_{s} \pounds t}}{\rho_{s} c_{Ps} k_{s} \widehat{W}_{0}} \sqrt{\frac{k_{s}}{2f}}, \\ D &= \bigg( 1 + \frac{\delta \sqrt{k_{s} \pounds t}}{D_{q}} + \frac{\sqrt{k_{s} \pounds t}}{D_{r}} \bigg), \\ D_{q} &= \frac{\rho_{s} c_{Ps} k_{s}}{L \rho k \gamma q_{s}} \bigg( \frac{\partial \ln \tilde{e}_{s}}{\partial T} \bigg)^{2}, \\ D_{r} &= \frac{\rho_{s} c_{Ps} k_{s}}{\rho c_{P} k} \sqrt{\frac{k_{s} \pounds t \tau_{R}}{\varDelta t + \tau_{R}}}. \end{split}$$

# **III. THE MONTHLY PREDICTION EXPERIMENTS**

One-month prediction for February to December, 1980 have been performed with the analogy-dynamical model. Numerical integrations are run for 30 days. The basic state is selected from historical data according to similarity criterion, the analogous formula is taken as

$$r=\frac{1}{2}(r_A+r_L),$$

where  $r_A$  the correlation coefficient of 500hPa anomaly field,  $r_L$  the correlation coefficient of carth's surface temperature anomaly field. The assessment of 500hPa anomaly field ( $\phi$ ) and earth's surface temperature anomaly field ( $T'_s$ ) predicted by analogy-dynamical model is shown in Table 1. It is can be found from Table 1 that the forecasts by this model are better than statistical analogous forecasts for most months, especially for atmospher. The

500hPa geopotential height anomalies predicted and observed for March. Junc. September 1980 are shown in Figs. 1,2,3 respectively.

Table 1.	Assessment of		500hPa Anomaly	Field $(\phi')$ and	Earth Surface	Temperature	Anomaly I	Field	(T')
	Predicted	by	Analogy-Dynamica	l Model					

		Feb.	March	Apr.	Мау	June	July	Aug.	Sept.	Oct.	Nov.	Dec.	Mean
¢'	ADP* SAF**		0.34 -0.31										
T's	ADP SAF	-0.24 0.3											1

\* ADP— the correlation coefficient between model predicted and observed values over the Northern Hemisphere.

\*\*SAF— the correlation coefficient between single analogous forecast and observed values over the Northern Hemisphere.



Fig. 1. Predicted (a) and observed (b) 500hPa geopotential height monthly anomaly field for March 1980.

By comparing Fig. 1a with Fig. 1b it can be seen that the regions with positive and negative predicted anomalies of 500hPa geopotential heights are similar to the observed ones except for eastern coast of South America, but the intensity of the predicted field is stronger than that of the observed field. This is due to the fact that predicted field is the deviation field superposed on historical analogous anomaly field. Comparison of Fig. 2a with Fig. 2b shows that the predicted field is different from the observed, particularly in polar region. It can be found from Fig. 3 that the predicted 500hPa height anomalous field is close to the observed one. Particularly, the positive and negative regions are in very agreement with observations.



.-----

Fig. 2. As in Fig. 1 but for June 1980.



Fig. 3. As in Fig. 1 but for September 1980.

# IV. CONCLUDING REMARKS

Preliminary results of monthly prediction experiment show that the analogy-dynamical model has certain capacity for monthly prediction. It can remedy partially the defects of model with information provided by historical data. These results of prediction experiment are encouraging. The analogy-dynamical model, after further improved, is hopeful to be used

for doing monthly prediction.

It is also seen from the experiments that the monthly predicted field still shows some disagreements from the observed field. These disagreements may be connected with the unrealistic parameterization of some physical processes and selection of historical analogous field. There may be some other causes for the disagreement, which need to be examined and improved by more experiments in future.

Finally, it should be pointed out that although the predicted skill is not so high, this approach on long-range numerical weather forecast is a promising way worthy of further study.

#### REFERENCES

Chao, J.P., Guo, Y.F. and Xin R.N. (1982). A theory and method of long-range numerical weather forecast, J. Meteor. Sci. of Japan, 60:, 282-291.

Huang Jianping (1988), Observational, theoretical and numerical studies for monthly mean circulation anomaly, Ph. D. Thesis, Department of Atmospheric Sciences, Lanzhou University.

Huang Jianping and Chou Jifan (1990), The studies on the analogous rhythm phenomena in coupled oceanatmosphere system, *Scientia Sinica B*, 33:851-860.

Kuo, H.L. (1968), On a simplified radiative-conductive heat transfer equation, Scientific Report, No. 14, The planetary circulation project, the University of Chicago.

Navarra, A. and Miyakoda, K. (1984), Anomaly general circulation models, Proc. of the Climate Diagnostics Workshop, Corvallis Oregon, NOAA, 257-261.

Wang Shaowu (1984), The rhythm in the atmosphere and oceans in application to long-range weather forecasting, Adv. Atmos. Sci., 1:7-18.