

Wavelet analysis of global warming

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1. INTRODUCTION

The close attention has been paid to the climatic change for its important influence on economy and society. Now, the warming of the global climate for the last one hundred years is one of the hot topics in the studies of the climatic change.^{5,6,7} It is generally acknowledged that the increasing of greenhouse gases, such as CO_2 , which enhanced the greenhouse effect, is responsible for the warming of the global climate in the 20th century, especially in the 1980s. In this paper, the global warming is reexamined using of the wavelet transform "microscope". Wavelet analysis is a mathematical technique introduced recently for seismic data and acoustic signals.⁴ It provides a two-dimensional unfolding of one-dimensional signals, resolving both the time and frequency as independent variable.²

2. THE WAVELET TRANSFORM

The wavelet transform consists of expanding functions over wavelets which are constructed from one single function g by means of dilatation and translation. The analysing wavelet g is a regular function. It is necessary, in order to perform wavelet transform, that the analysing wavelet should be localized around $x = 0$ and have a vanishing integral. From the single analysing function g we construct a set of wavelet functions³

$$g_{a,b}(x) = \frac{1}{a}g\left(\frac{x-b}{a}\right) \quad (1)$$

by translations and dilatations. The wavelet $g(x)$ is then used as "mathematical microscopes", in which g characterizes the optics, while b is the position analysed and $\frac{1}{a}$ the magnification. The parameter a , which has the length dimension, will henceforth be called the (analysing) scale. The wavelet transform of $s(x)$ is defined as¹

$$T_g(a,b) = \frac{1}{\sqrt{a}} \int_{-\infty}^{+\infty} s(x)g\left(\frac{x-b}{a}\right)dx \quad (2)$$

In fact, $T_g(a,b)$ is simply the convolution product of the signal and the scale analysing function g . The wavelet transform may also be regarded as local Fourier transform: for a given position b , the amplitude of T_g will be largest when the scale a is roughly equal to the inverse local wavenumber of the signal. In this paper we use a real wavelet g of Gaussian type: $g(x) = (1-x^2)\exp(-x^2/2)$, called the Mexican hat to investigate temporal variation of the global temperature.

3. THE VARIATION OF THE GLOBAL TEMPERATURE

The basic data used in this study is the series of Hansen's global, northern hemisphere and southern hemisphere annual temperature from 1880-1990.⁹ Figure 1 shows the temperature anomalies for the globe from 1880 to 1990. The linear trend of temperature increasing from the 80s of the 19th century to the 80s of the 20th century is straightforward. Before 1920s, most of the temperature anomalies were negative, since then the positive anomalies predominated. The increment of 10-year mean global average temperature from 1880s to 1980s was $0.55^\circ C$. However, Figure 1 also shows that the warming found for the last one hundred years was made up by a few jumps or abrupt changes rather than by a consistent and homogeneous increasing of temperature. This can be checked up by using of t-test.⁸ The temperatures are characterized with three peaks around 1900, 1940, 1985. The increasing of global temperature concentrated in a few short time intervals, the temperature among them decreased more or less gently or went up and down around some intermediate value. These characteristics also can be found in the temperature series of the northern and southern hemisphere, but are less clear over the southern hemisphere than over northern hemisphere (Figures omitted).

4. WAVELET ANALYSIS

To confirm the characteristics found above, the series of Hansen are analysed by using wavelet transform. Figure 2(a,b,c) show an overview of the wavelet analysis of the global, northern hemisphere and southern hemisphere temperature. The abscissa is the position b , which correspond to the time. The ordinate is the scale a , which corresponds to the period length which increases with increasing of a . The successive pitchfork branchings are observed when increasing the magnification($1/a$) which illustrates how the frequency changes with time. The position of the local singularities of the measure is easily identified as spatial point b at different scale a . Compared Figure 2b with Figure 2c, it can be found that the warming over the northern hemisphere is stronger than over the southern hemisphere.

Short time scale pattern is revealed in Figure 3a, where the wavelet analysis is performed at the scale $a = l_0^{11}$. This structure seems to be nature climate fluctuation in this time scale range. The main mechanism forming this fluctuation may be quasi- biennial oscillation(QBO) phenomenon. Very different long time scale pattern is displayed in Figure 3b, which represents the wavelet transform at scale $a = l_0^{70}$. This pattern is created by three abrupt warming process. The first occurred between the 80s and 90s of the 19th century. The second appeared during the 20s and 30s of the 20th. The third occurred near 1980. These results are similar with Yi and Wang's results. It provides new evidences of abrupt warming. The linear trend of the global warming from the 80s of the 19th century to the 80s of the 20th century are shown in Figure 3c, which represents the wavelet transform at scale $a = l_0^{120}$.

5.CONCLUSION

As we have seen in above figures, climate variation displayed multiple scale features are created by different physics processes. The climate oscillation may be the interaction between subclimate systems. The global climate system is a complicate nonlinear dynamic system which is composed of multiple scale system and are characterised by bifurcating and abrupt dynamical behavior. Moreover, The climate jump may raise from out factor, such as volcano and sun activity and so on, and the linear warming may be from the greenhouse effect.

The results show that the wavelet analysis is a powerful tool for analysising the climate change. It provides a very efficient method for characterizing not only the static situation but also the dynamical phenomena. Its application to meteorology looks very promising.

6.REFERENCES

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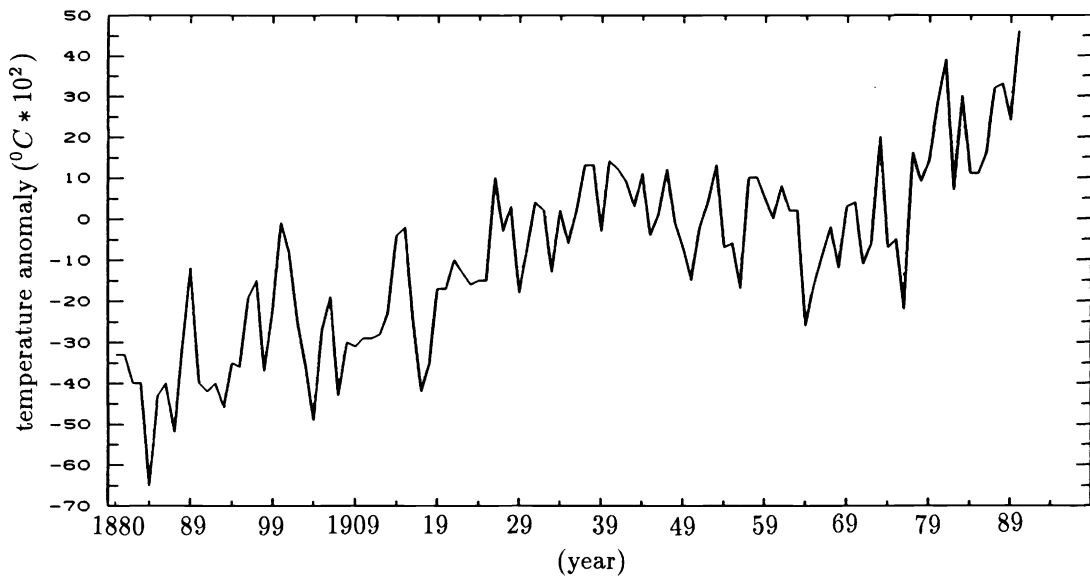
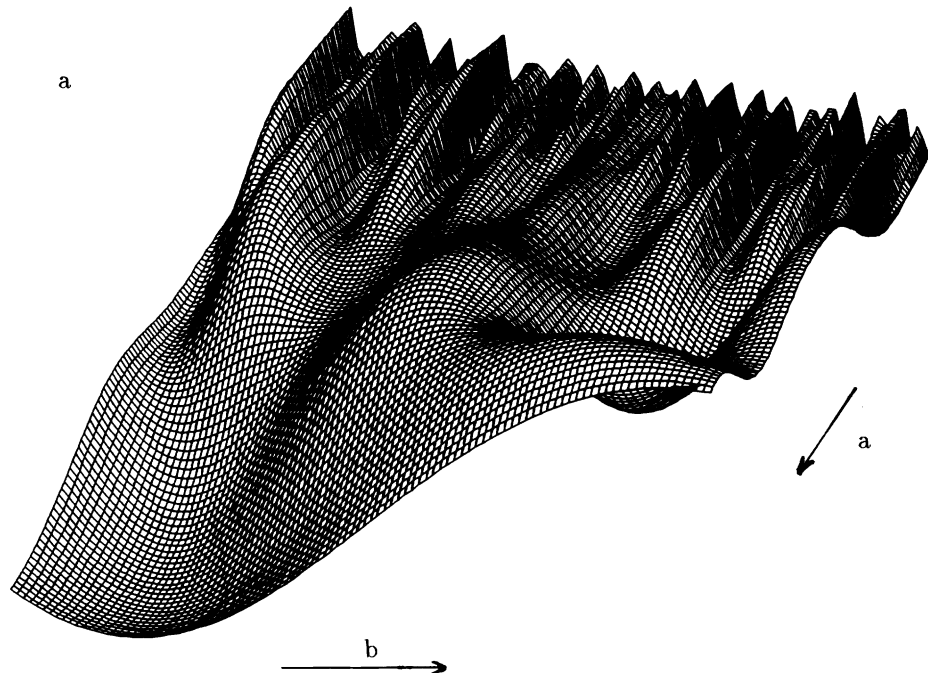


Figure 1: The temperature anomalies for the globe from 1880 to 1990 (relative to a 1951-1981 reference period).



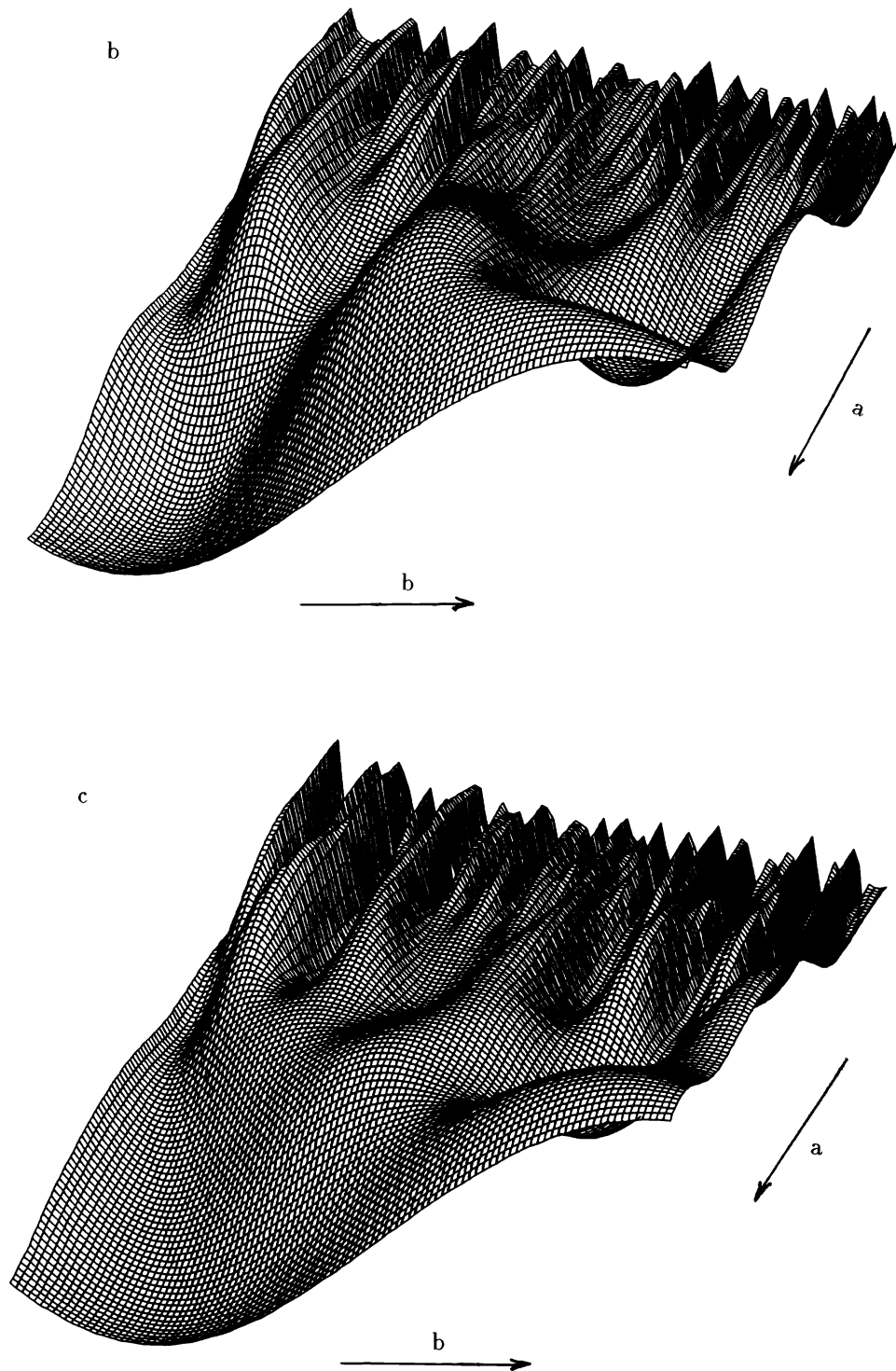


Figure 2: The wavelet transform of the temperature anomalies, a) for the globe; b) for the Northern Hemisphere; c) for the Southern Hemisphere.

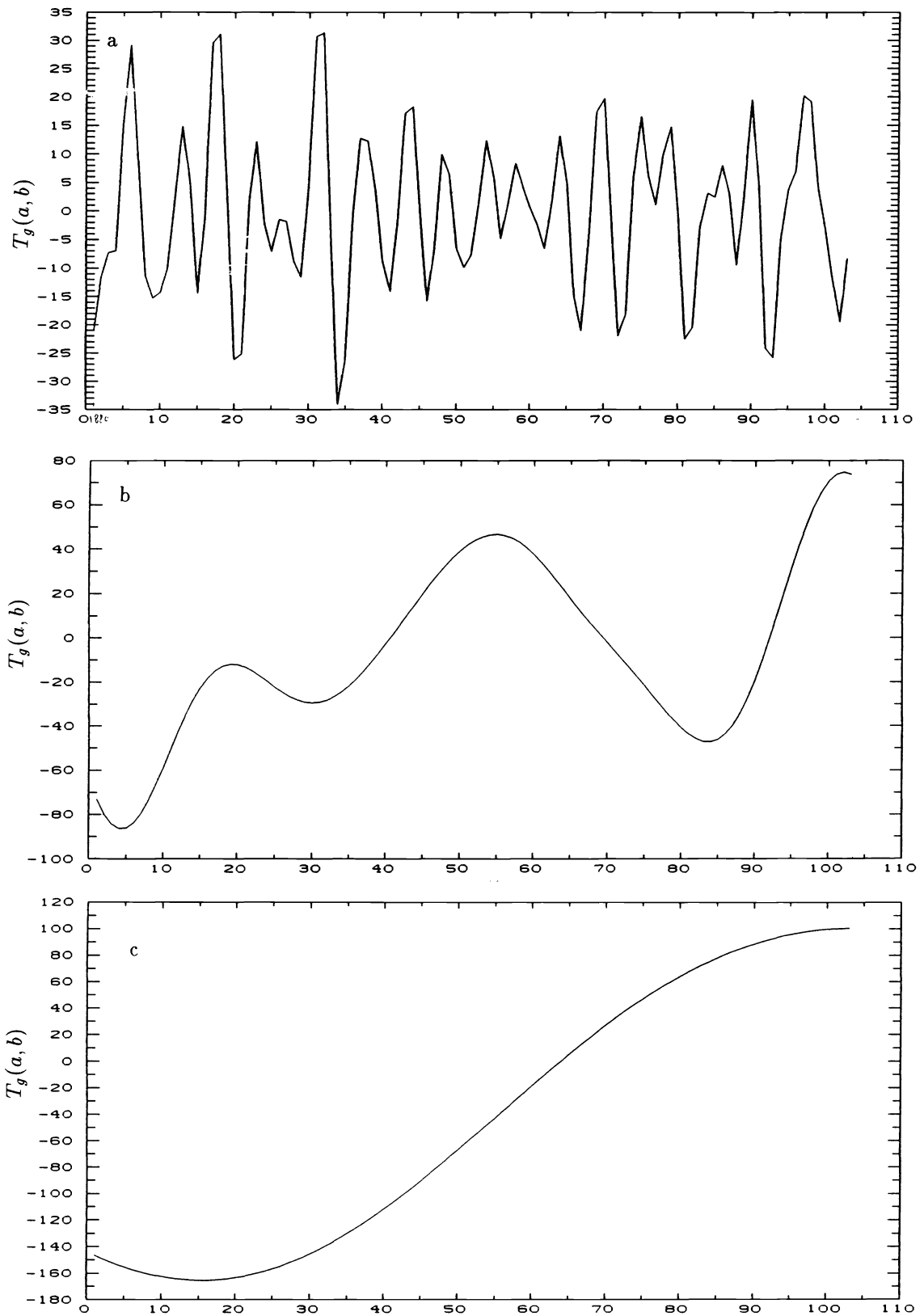


Figure 3: The wavelet transform, a) at scale $a = l_0^{11}$; b) at scale $a = l_0^{70}$; c) at scale $a = l_0^{120}$.